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The influence of external electromagnetic field on an exciton spin current

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Abstract
A pure spin current of a triplet exciton has been proposed for a three-dimensional system based on Dirac relativistic quantum mechanics, considering spin to be an intrinsic relativistic quantum variable. The transport dissipations of the triplet exciton caused by the magnetic excitation and the Coulomb interaction (such as the bremsstrahlung effect) can be minimized, making it possible to produce a pure exciton spin current with minimized charge-induced dissipation for the transport of information in spin-based devices. The exciton spin current can be expressed as a sum of an electron spin current and a hole spin current, modulated by an envelope function. It flows along the tangential direction of the isodense of the exciton. Both external electric and magnetic fields influence the exciton spin current and can be used to separate the triplet exciton and the singlet exciton. A simplification of our model, i.e., considering only an external electric field, yields the same result reported by Shen (2005 *Phys. Rev. Lett.* **95** 187203), in which the spin current was phenomenologically derived by the expectation value of the product of spin and velocity observables. Incorporation and possible application of the exciton spin current in several systems are discussed.

Spintronics is a subdiscipline of condensed matter physics whose main aim is to control and manipulate the electronic spin degree of freedom. The primary attention focuses on the basic physical principles underlying the spin injection, spin polarization, spin dynamics, and spin-polarized transport in semiconductors and metals [1–3]. Many proposed spintronic device schemes turn out to be practical eventually. At present, spin injection has been successfully realized in diluted magnetic semiconductors (DMSs) [4, 5]. Nevertheless, the Curie temperature of the DMSs is still low and unstable for practical use at room temperature. Apart from the DMSs that make use of both the charge and spin of electrons to process and

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store information simultaneously, there is an alternative for spin injection, i.e., the electron spin degree of freedom entirely replaces the charge degree of freedom as the carrier of information. Compared with the charge, the spin has superior advantages. It can be easily manipulated by an external magnetic field. On the other hand, the coherent length and the relaxation time of the spin are so long that once it is generated, it can last for a long time, while the charge state is easily destroyed by scattering and collision with defects, impurities and even other charges. It is possible to produce smaller and more powerful devices by making use of the spin of the electron.

It is well known that a charge current describes the transport of charges in conventional electronics. Correspondingly, there is a spin current that can describe the spin transport in spintronics. So far, several definitions of spin current have been proposed. It was conventional to define the spin current simply as the expectation value of the product of spin and velocity observables [6]. A slightly improved definition of the spin current was given by the time derivative of the spin displacement that was the product of spin and position observables [7]. However, these definitions are combined into a semiclassical spin continuity equation that describes a mean-value quantity of the whole system, but not a local quantity which is expected to differ from point to point in space [8]. Another definition was phenomenologically generalized from the transport equation of the charge current [9]. Considering that the electron spin is an intrinsic relativistic quantum variable, it is necessary to study the spin current based on relativistic quantum mechanics.

The spin current usually flows with an accompanied charge current. In the process of charge transport, the effect of bremsstrahlung radiation leads to a dissipation of energy. It is desired to generate a pure spin current that can dissipationlessly transport information in a three-dimensional system. Recently, Zhang *et al* generalized the quantum Hall effect from two to four spatial dimensions and predicted a purely topological and dissipationless spin current at room temperature at the boundary of the four spatial dimensions by an electric field [9, 10]. Note the fact that the presence of the electric field leads to charge dissipation, and the quantum Hall effect exists only at extremely low temperatures. It is reasonable to argue whether such a spin current could be dissipationless in an electric field at room temperature. Another dissipationless spin current originated from a two-dimensional electron system with Rashba spin–orbit coupling [11]. To some extent, this is not a complete dissipationless spin current if the dissipation is caused by the charge current transport. Shen [6] found that an electric field exerted a transverse force on a moving electron spin, and the spin current was conventionally defined in a mean-value form. This model, however, cannot be used to describe the local changes. Stevens *et al* [12] demonstrated all-optical quantum interference injection and the control of a ballistic pure spin current (without an accompanying charge current) in GaAs/AlGaAs quantum wells, consisting of spin-up electrons traveling in one direction and spin-down electrons traveling in the opposite direction.

In this study, we propose an intrinsically pure spin current of triplet exciton for a three-dimensional system based on the Dirac relativistic quantum mechanics, considering that the exciton universally exists in wide band gap semiconductors, and is stable even at room temperature (e.g. the exciton of ZnO). Because both the total magnetic moment and the charge of the triplet exciton equal zero, while the total angular momentum of the spin does not equal zero, the transport dissipations due to magnetic excitation and Coulomb interaction (such as the bremsstrahlung effect) can be minimized. In this way, a pure spin current of triplet excitons can be generated. Such an exciton spin current is deduced from the continuity equation of Dirac theory based on the fact that the spin is an intrinsic relativistic quantum variable. The transport property of the exciton spin current in an external electromagnetic field is studied, and possible applications are discussed.

We start with the equation of continuity of the Dirac fields [13]:

$$\partial_\mu j^\mu = 0, \tag{1}$$

where $j^\mu = \bar{\psi}\gamma^\mu\psi = (\psi^*\psi, \psi^*\gamma^0\gamma\psi) = \psi^*\alpha\psi$ is the four-vector current density. α and γ are 4×4 Dirac matrices. The four-component quantity ψ, which is the wavefunction for an electron or a hole, can be taken as a function of two-component quantities ϕ and χ by $\psi = \binom{\phi}{\chi}$. Thus, the current density for one particle becomes

$$\vec{j} = \psi^*\alpha\psi = \phi^*\vec{\sigma}\chi + \chi^*\vec{\sigma}\phi, \tag{2}$$

where $\vec{\sigma}$ are the Pauli matrices. This term is obtained by expansions in powers of $1/c$ and $1/c^2$ (c is the velocity of light) and effected by the well-known procedure of expressing the small component χ of ψ in terms of its large component ϕ. According to the Pauli equation, the first approximation (with respect to $1/c$) for the current density of one particle in an external electromagnetic field is given by

$$\vec{j} = \frac{\mathrm{i}\hbar}{2m}(\phi\nabla\phi^* - \phi^*\nabla\phi) - \frac{e}{mc}\vec{A}\phi^*\phi + \frac{\hbar}{2m}\nabla\times(\phi^*\vec{\sigma}\phi), \tag{3}$$

where the first term is the conventional transport current, and the second term is relevant to the Meissner effect in a superconductor. Only the last term involves a spin factor, from which the spin-related current is introduced.

Continuing the expansion to the order of $1/c^2$ and substituting the second approximate expression of χ,

$$\chi = \frac{1}{2mc}\left(1 - \frac{\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi}{2mc^2}\right)\vec{\sigma}\cdot\left(-\mathrm{i}\hbar\nabla - \frac{e}{c}\vec{A}\right)\phi, \tag{4}$$

into (2) yields

$$\vec{j} = \left(1 - \frac{\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi}{2mc^2}\right)\left[\frac{\mathrm{i}\hbar}{2m}(\phi\nabla\phi^* - \phi^*\nabla\phi) - \frac{e}{mc}\vec{A}\phi^*\phi + \frac{\hbar}{2m}\nabla\times(\phi^*\vec{\sigma}\phi)\right], \tag{5}$$

where $\vec{A}$ is the vector potential, Φ the scalar potential, and the operator $-\mathrm{i}\hbar\partial/\partial t$ only acts on the wavefunction. The first term in equation (5) describes the conventional transport current in quantum mechanics with the relativistic correction $\left(1 - \frac{\mathrm{i}\hbar\partial/\partial t - e\Phi}{2mc^2}\right)$. The second term can be ignored because of the absence of spin. The last term is a current resulting from the spin that we are interested in, and can be expressed as

$$\vec{j}^\sigma = \left(1 - \frac{\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi}{2mc^2}\right)\frac{\hbar}{2m}\nabla\times(\phi^*\vec{\sigma}\phi). \tag{6}$$

It can be seen from equation (6) that the spin-related current is intrinsic and different from the reported phenomenological definitions in an electric field [9, 10, 14]. Meanwhile, this spin-related current $\vec{j}^\sigma$ is a local quantity, and it changes at every point in space. In addition, $\vec{j}^\sigma$ depends on the energy $\frac{\mathrm{i}\hbar\partial/\partial t - e\Phi}{2mc^2}$, and its direction is determined by the gradient of the spin probability. If the spin direction is projected along the z-axis, it flows in the plane perpendicular to the z-axis. On the other hand, the factor $\phi^*\vec{\sigma}\phi$ is considered as an axial vector density. Therefore, $\nabla \times (\phi^*\vec{\sigma}\phi)$ would generate a closed loop like a closed magnetic circuit. It is predicted that this term forms a closed spin circuit that can transport information carried by the spins.

In order to introduce an intrinsically pure spin current in a three-dimensional system in which spin–charge separation is generally thought impossible, a spin-polarized exciton is first considered. Note that the flow of an exciton with a triplet state in which both the electron and the hole have the same spin direction does not form an electric current due to the opposite charge of the electron and the hole. Since the total magnetic moment and charge of the triplet exciton equal zero, the transport dissipation due to magnetic excitation and Coulomb interaction (such as the bremsstrahlung effect) can be minimized. Meanwhile, the spin-polarized exciton is stable even in an external electromagnetic field, because the triplet state of an exciton has a much longer lifetime than the singlet state (with opposite spin direction). In a sense, it may generate a pure spin current of triplet excitons. The so-called Wannier excitons are indeed excitons existing in semiconductors. The exciton wavefunction can be expressed by the linear combination of the product of the electron wavefunction and the hole wavefunction [15]:

$$\phi^{\mathrm{nlm},\vec{K}}(\vec{r}_{\mathrm{e}},\vec{r}_{\mathrm{h}}) = \Omega^{-1/2}\mathrm{e}^{\mathrm{i}\vec{K}\cdot\vec{R}}\phi^{\mathrm{env}}_{\mathrm{nlm}}(\vec{r}_{\mathrm{e}}-\vec{r}_{\mathrm{h}})\phi_{\mathrm{e}}(\vec{r}_{\mathrm{e}})\phi_{\mathrm{h}}(\vec{r}_{\mathrm{h}}), \tag{7}$$

where $\Omega^{-1/2}$ is the normalization factor, $\vec{K}$ is the total wavevector and $\vec{R}$ is the radius vector of the center of mass. The plane-wavevector in equation (7) describes the free propagation of a Wannier exciton through the periodic lattice, and the hydrogen-atom-like envelope function $\phi^{\mathrm{env}}_{\mathrm{nlm}}$ gives the relative motion of the electron and the hole. The wavefunctions for the electron and the hole can be given in the same simplified form of the Wannier function including the spin wavefunction:

$$\phi_{\mathrm{e}}(\vec{r}_{\mathrm{e}}) = u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})\chi(\sigma_{\mathrm{e}}), \qquad \phi_{\mathrm{h}}(\vec{r}_{\mathrm{h}}) = u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})\chi(\sigma_{\mathrm{h}}). \tag{8}$$

Therefore, the exciton spin current according to equation (6), can be written as

$$\begin{aligned}\vec{j}^{\sigma}{}_{\mathrm{exciton}} &= \left(1-\frac{\mathrm{i}\hbar\frac{\partial}{\partial t}-e\Phi}{2mc^{2}}\right)\frac{\hbar\Omega}{2}\phi^{*\,\mathrm{env}}_{\mathrm{nlm}}(\vec{r}_{\mathrm{e}}-\vec{r}_{\mathrm{h}})\phi^{\mathrm{env}}_{\mathrm{nlm}}(\vec{r}_{\mathrm{e}}-\vec{r}_{\mathrm{h}})\\ &\quad\times\left\{\frac{1}{m_{\mathrm{e}}}\nabla_{\mathrm{e}}\times[\phi^{*}_{\mathrm{e}}(\vec{r}_{\mathrm{e}})\vec{\sigma}\phi_{\mathrm{e}}(\vec{r}_{\mathrm{e}})]+\frac{1}{m_{\mathrm{h}}}\nabla_{\mathrm{h}}\times[\phi^{*}_{\mathrm{h}}(\vec{r}_{\mathrm{h}})\vec{\sigma}\phi_{\mathrm{h}}(\vec{r}_{\mathrm{h}})]\right\}.\end{aligned} \tag{9}$$

This indicates that the pure spin current of the exciton, which is the sum of the electron spin current and the hole spin current, is modulated by the envelope function. In equation (9), the nabla operators act on the envelope functions; however, it can be eliminated for the summation of the electron and the hole. If we set the spin direction along the z-axis with the eigenfunction $\chi_{\mathrm{e,h}} = \binom{1}{0}$, the last terms in equation (9) become

$$\begin{aligned}&\frac{1}{m_{\mathrm{e}}}\nabla_{\mathrm{e}}\times[\phi^{*}_{\mathrm{e}}(\vec{r}_{\mathrm{e}})\vec{\sigma}\phi_{\mathrm{e}}(\vec{r}_{\mathrm{e}})]+\frac{1}{m_{\mathrm{h}}}\nabla_{\mathrm{h}}\times[\phi^{*}_{\mathrm{h}}(\vec{r}_{\mathrm{h}})\vec{\sigma}\phi_{\mathrm{h}}(\vec{r}_{\mathrm{h}})]\\ &\quad=\frac{1}{m_{\mathrm{e}}}\nabla_{\mathrm{e}}\times[u^{*}_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})\chi^{+}_{\mathrm{e}}\vec{\sigma}\chi_{\mathrm{e}}]+\frac{1}{m_{\mathrm{h}}}\nabla_{\mathrm{h}}\times[u^{*}_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})\chi^{+}_{\mathrm{h}}\vec{\sigma}\chi_{\mathrm{h}}]\\ &\quad=\frac{1}{m_{\mathrm{e}}}[\vec{i}\,\partial_{y}(u^{*}_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}}))-\vec{j}\,\partial_{x}(u^{*}_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}}))]\\ &\qquad+\frac{1}{m_{\mathrm{h}}}[\vec{i}\,\partial_{y}(u^{*}_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}}))-\vec{j}\,\partial_{x}(u^{*}_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}}))].\end{aligned} \tag{10}$$

In the case of the tight-binding approximation, the Wannier functions $u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}})$ and $u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}})$ in equation (10) are given by [16]

$$\begin{aligned}u_{\vec{R}_{\mathrm{e}}}(\vec{r}_{\mathrm{e}}) &= a(\vec{r}_{\mathrm{e}}-\vec{R}_{\mathrm{e}})-\tfrac{1}{2}\sum_{\vec{d}_i}S(\vec{d}_{\mathrm{i}})a(\vec{r}_{\mathrm{e}}-\vec{R}_{\mathrm{e}}-\vec{d}_{\mathrm{i}})\\ u_{\vec{R}_{\mathrm{h}}}(\vec{r}_{\mathrm{h}}) &= a(\vec{r}_{\mathrm{h}}-\vec{R}_{\mathrm{h}})-\tfrac{1}{2}\sum_{\vec{d}_i}S(\vec{d}_{\mathrm{i}})a(\vec{r}_{\mathrm{h}}-\vec{R}_{\mathrm{h}}-\vec{d}_{\mathrm{i}}),\end{aligned}$$

where $a(\vec{r}_{\mathrm{e}}-\vec{R}_{\mathrm{e}})$ and $a(\vec{r}_{\mathrm{h}}-\vec{R}_{\mathrm{h}})$ are the atomic wavefunctions and $S(\vec{d}_{\mathrm{i}})$ is the overlap integral.

Equation (10) shows that the exciton spin current flows in the plane perpendicular to the spin-polarized direction. The direction of the exciton spin current is along the tangential direction of the isodense of the exciton.

Taking ZnO as an example, it is known that excitons of ZnO are thermally stable at room temperature, and the lifetime could increase from nanoseconds to beyond milliseconds [17]. Therefore, a pure spin current of triplet excitons in ZnO may exist at room temperature. For a single-domain ferromagnetic or antiferromagnetic nanoparticle, the exciton spin current flows along the isodense of the particle and can be used to transport information.

In order to study the transport property of the pure exciton spin current in an electromagnetic field, the time derivative of the exciton spin current $\vec{j}^{\sigma}{}_{\text{exciton}}$ is carried out:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{j}^{\sigma}{}_{\text{exciton}}(\vec{r},t) = \frac{\Omega}{\hbar}\phi^{*\,\text{env}}_{\text{nlm}}(\vec{r}_{\text{e}}-\vec{r}_{\text{h}})\phi^{\text{env}}_{\text{nlm}}(\vec{r}_{\text{e}}-\vec{r}_{\text{h}})\left\{\frac{1}{m_{\text{e}}}(\phi^{*}_{\text{e}}(\vec{r},t)[\hat{F},\hat{H}]\phi_{\text{e}}(\vec{r},t)) + \frac{1}{m_{\text{h}}}(\phi^{*}_{\text{h}}(\vec{r},t)[\hat{F},\hat{H}]\phi_{\text{h}}(\vec{r},t))\right\}, \tag{11}$$

according to the Heisenberg equation $\frac{\mathrm{d}}{\mathrm{d}t}\hat{o} = \frac{1}{\mathrm{i}\hbar}[\hat{o},\hat{H}]$. The operator $\hat{F}$ in equation (11) is expressed as

$$\hat{F} = \left(1 - \frac{\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi}{2mc^2}\right)\vec{p}\times\vec{\sigma}\left(1 - \frac{(\vec{p}-\frac{e}{c}\vec{A})^2}{4m^2c^2}\right). \tag{12}$$

In the derivation below, one uses the following Hamiltonian:

$$\hat{H} = \hat{H}^{(2)} = \frac{(\vec{p}-\frac{e}{c}\vec{A})^2}{2m} - \frac{\vec{p}^{\,4}}{8m^3c^2} + e\Phi - \frac{e\hbar}{2mc}\vec{\sigma}\cdot\vec{H} - \frac{e\hbar^2}{8m^2c^2}\nabla\cdot\vec{E} - \frac{e\hbar}{4m^2c^2}\vec{\sigma}\cdot\left[\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right)\right]. \tag{13}$$

This is the second approximate Hamiltonian considering the relativistic effect in the presence of an external electromagnetic field. The first term denotes a kinetic energy, and the second is due to the relativistic correction of the kinetic energy. The third term, $e\Phi$, is a static electronic energy in an electrostatic field. The fourth term presents the interaction between the magnetic moment and the external magnetic field, indicating that an electron has an intrinsic magnetic moment and there is a potential $-\vec{\mu}\cdot\vec{H}$ in the external magnetic field. The fifth term in equation (13) is related to the Darwin term, while the last one is attributed to the energy of the interaction between the moving magnetic moment and the electric field. The relative moving electric field yields a magnetic field $\vec{H}_{\text{intrinsic}} = \vec{E}\times\vec{v}/c$, in which the energy of the magnetic moment should be $-\vec{\mu}\cdot\vec{H}_{\text{intrinsic}}$. In the case of a central Coulombic field, the last term becomes

$$-\frac{e}{4m^2c^2}\vec{\sigma}\cdot\left[\frac{1}{r^3}\vec{r}\times\vec{p}\right] = -\frac{e}{4m^2c^2}\frac{1}{r^3}\vec{\sigma}\cdot\vec{L},$$

which is assigned to the spin–orbit coupling.

The change rate of the exciton spin current $\vec{j}^{\sigma}{}_{\text{exciton}}$ with time depends on $[\hat{F},\hat{H}]$. It can be expressed as

$$[\hat{F},\hat{H}] = (2\nabla\vec{A}\cdot\nabla + \nabla^2\vec{A})\times\vec{\sigma}\left(\frac{e\hbar^2}{2mc} - \frac{(\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi)e\hbar^2}{4m^2c^3} - \frac{(\vec{p}-\frac{e}{c}\vec{A})^2e\hbar^2}{8m^3c^3}\right) + \mathrm{i}\hbar e\vec{E}\times\vec{\sigma}\left(1 - \frac{(\mathrm{i}\hbar\frac{\partial}{\partial t} - e\Phi)}{2mc^2} - \frac{(\vec{p}-\frac{e}{c}\vec{A})^2}{4m^2c^2}\right) + \frac{\mathrm{i}\hbar^3e}{8m^2c^2}\nabla^2\vec{E}\times\vec{\sigma}$$

$$
\begin{aligned}
&+ \mathrm{i}\hbar^2 e(\vec{\sigma}\cdot\nabla\vec{H})\times\vec{\sigma}\left(\frac{1}{2mc}-\frac{(\mathrm{i}\hbar\frac{\partial}{\partial t}-e\Phi)}{4m^2c^3}-\frac{(\vec{p}-\frac{e}{c}\vec{A})^2}{8m^3c^3}\right)\\
&-\frac{\mathrm{i}\hbar e}{4m^2c^2}\left\{\nabla^2\Phi-2\vec{E}\cdot\nabla+2\mathrm{i}\hbar\frac{e}{c}\vec{A}\cdot\vec{E}\right\}\nabla\times\vec{\sigma}\\
&-\frac{\mathrm{i}\hbar e}{4m^2c^2}\nabla\times\vec{\sigma}\left\{\nabla^2\Phi-2\vec{E}\cdot\nabla+2\mathrm{i}\hbar\frac{e}{c}\vec{A}\cdot\vec{E}\right\}\\
&-\mathrm{i}\hbar^2 e\nabla\times[\vec{\sigma},\vec{\sigma}\cdot\vec{H}]\left(-\frac{1}{2mc}+\frac{(\mathrm{i}\hbar\frac{\partial}{\partial t}-e\Phi)}{4m^2c^3}+\frac{(\vec{p}-\frac{e}{c}\vec{A})^2}{8m^3c^3}\right)\\
&-\mathrm{i}\hbar^2 e\nabla\times\left[\vec{\sigma},\vec{\sigma}\cdot\left(\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right)\right)\right]\left(-\frac{1}{4m^2c^2}+\frac{(\mathrm{i}\hbar\frac{\partial}{\partial t}-e\Phi)}{8m^3c^3}\right).
\end{aligned}
$$

For a stationary state, the operator $-\mathrm{i}\hbar\partial/\partial t$ can be replaced by an energy of ε. In this study, however, the operator $-\mathrm{i}\hbar\partial/\partial t$ in equation (12) is replaced by the Hamiltonian $\hat{H}^{(2)}$ for universal applications according to the Dirac equation in the second approximation, $\mathrm{i}\hbar\frac{\partial}{\partial t}\phi = \hat{H}^{(2)}\phi$. Obviously, the expanded expression for this commutator is very complicated. Terms of the expansion of higher orders have been ignored. In the case of an exciton, most of the previous works focused on the relative motions of the electron and the hole which belong to the internal motion for the exciton. In this work, however, we focus on the integral motion of the exciton. Therefore, only the terms involving an even power of e in equation (11) are considered, since the other terms deform the relative motion of the exciton by the Lorentz force [15] and do not affect the center of mass of the exciton. In order to understand the influence of an external field on the motion of the exciton spin current, the terms involving $\vec{\sigma}$, $\vec{E}$, and $\vec{H}$ remain to be discussed. They are given as follows:

$$
\begin{aligned}
[\hat{F},\hat{H}] \Rightarrow\ &\frac{\mathrm{i}\hbar^2e^2}{4m^2c^3}\vec{E}\times\vec{\sigma}\left\{\vec{\sigma}\cdot\left[\vec{H}+\frac{1}{2mc}\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right)\right]\right\}\\
&+\frac{e^2\hbar^2}{2m^2c^3}\left\{\left(\vec{A}\cdot\left(\vec{E}-\frac{1}{c}\frac{\partial}{\partial t}\vec{A}\right)\right)\nabla\times\vec{\sigma}+\nabla\times\vec{\sigma}\left(\vec{A}\cdot\left(\vec{E}-\frac{1}{c}\frac{\partial}{\partial t}\vec{A}\right)\right)\right\}\\
&-\frac{e^2\hbar^3}{2m^3c^4}\left\{\left(\vec{\sigma}\cdot\left[\vec{H}+\frac{1}{2mc}\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right)\right]\right)\right.\\
&\left.\times\vec{\sigma}\cdot\left(\nabla\left[\vec{H}+\frac{1}{2mc}\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right)\right]\right)\right\}.
\end{aligned}
\tag{14}
$$

The factors $\hbar$ and $1/c^3$, $1/c^4$ in the above equation imply a quantum mechanism and relativistic effects, respectively. It is important to point out that the first and the third terms involve an effective magnetic field, namely,

$$\vec{H}_{\mathrm{eff}} = \vec{H}+\frac{1}{2mc}\vec{E}\times\left(\vec{p}-\frac{e}{c}\vec{A}\right), \tag{15}$$

where $\vec{H}$ is the external magnetic field, and the other term is caused by the relatively moving electric field. It can be seen that both extrinsic and intrinsic magnetic fields exist in our model.

It is important to understand the physical meanings of the terms in equation (14). The first term contains not only even powers of e, but also even powers of σ, representing an accelerated spin force that is perpendicular to the external electric field $\vec{E}$ and relevant to the effective magnetic field $\vec{H}_{\mathrm{eff}}$. The force depends on the polarized direction of the spin and the coupling energy of the magnetic moment, $\vec{\sigma}\cdot\vec{H}_{\mathrm{eff}}$. It is proposed that the motion of a pure exciton spin current can be accelerated under the influence of an electromagnetic field. If the external magnetic field $\vec{H}$ is absent, the force is perpendicular to the electric field and the velocity, resulting in the same results as reported in [6] in which the spin current and the spin force were mean values of the whole system and could not describe the spin force exerted by the field at every point of space. If the external electric field is absent, the last term in equation (14) becomes $-\frac{e^2\hbar^3}{2m^3c^4}\sigma_z^2 H_z(\partial_z\vec{H})$, driving the triplet exciton to move to the place with the smallest magnetic intensity in a non-uniform magnetic field. A singlet exciton that has a magnetic dipole, however, will move towards the place with the strongest magnetic intensity in a non-uniform magnetic field. In this way, a singlet exciton and a triplet exciton can be separated. From the discussion above, it is concluded that the motion of a pure spin current of a triplet exciton can be controlled by an external field, which makes it possible to transport information carried by the spin without any charge-induced dissipation.

It should be pointed out that the proposed model can be applied to positronium [18, 19] in which two states coexist. Parapositronium (singlet state) has a short lifetime without a spin, while orthopositronium (triplet state) has a much longer lifetime with a spin of 1. Since the spontaneous radiation of the short-lived singlet state influences the simulated radiation of the triplet state, it is desired to separate them. According to our model, they can be separated because of their different spins by applying an external field. In this way, orthopositronium (triplet state) with longer lifetime can be collected for applications.

In conclusion, we have proposed a model to generate an intrinsically pure exciton spin current in a three-dimensional system. The exciton spin current is deduced by the relativistic quantum theory starting from the continuity equation of Dirac theory. The spin current of the triplet exciton flows along the tangential direction of the isodense of the exciton in the plane perpendicular to the spin-polarized direction. The transport property of the exciton spin current in an electromagnetic field has been investigated. It has been found that the motion of the exciton spin current can be controlled by the external field, and thus, the transport of information carried by the spin can be controlled. It is predicted that a pure spin current of triplet excitons in ZnO may exist at room temperature, and this model can also be applied to positronium to collect orthopositronium.

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